LEARNING GEOMETRY THROUGH DYNAMIC GEOMETRY SOFTWARE

Sue Forsythe investigates effective teaching and learning of geometrical concepts using dynamic geometry software.

It must be over 14 years since I first started taking classes into the computer room to work on programs such as LOGO and SMILE. I have always found that pupils enjoyed these lessons and were learning in a different way from how they did in the classroom. However, I was often disappointed that the pupils did not transfer skills learned at the computer to their traditional pen and paper tasks in the classroom. Despite this drawback, I felt that pupils had an enormous amount to gain from integrating computers into education. I also felt that computers are a large part of young peoples' lives and that school will be seen as insignificant and backwards if it is not up-to-date with the rest of the world in using ICT.

I recently carried out a project that has been my journey towards a better understanding of how we can learn in an environment that includes the computer (Forsythe, 2006). In particular, I have considered the sub-environment of dynamic geometry software (DGS). The reason for this choice is that I believe that programs like Geometer's Sketchpad and Cabri are truly interactive; ie, there is some form of communication between the student and the computer which enables the student to go further with the mathematics than they would otherwise have been able to. The computer can only obey inputted instructions, and so the student must be precise about what he or she inputs. Therefore the student is forced to think clearly and logically (Goldstein, 2002) about their inputs and about the responses from the computer. When this happens, it is possible for the student plus computer to achieve more than the student could on their own.

The DGS environment

When the student works on DGS, many different things may be happening, but I want to consider two elements in particular:

1 The medium of the computer

The computer plus software is a medium through which learning is taking place, and it is important not to underestimate the effect that this medium has on the student relative to the pencil and paper medium, the text book medium, television as a medium, and so on (Salomon, 1994). In order to be able to learn in this situation, the student needs to be familiar with the medium of the computer and be familiar with the software being used. Otherwise, the problems of using the computer will detract from the geometry problems the teacher was intending the student to think about.

2 Computer-generated geometry versus paper geometry

The second issue is the form of the mathematics on the computer. For instance, on paper, a geometrical object such as a triangle is static. It is formed from three straight-line segments of fixed length with fixed angles between them and a fixed position on the paper. It is one example of a triangle among an infinity of triangles. However, a triangle on a computer screen constructed using DGS is rather different. It will not be a static triangle fixed in space, and how it behaves will depend on the method used to construct it on the screen (Olive, 2000). It is my observation that a student is able to learn more about the principles
of geometry through the triangle constructed using DGS than through the one drawn on paper.

The dynamic geometry environment has essentially produced a new kind of geometry. In this computer geometry, the figure is determined by the construction process and how it then behaves under dragging. Thus, the same geometrical figure can give rise to two different software-generated figures (Laborde, 1993). This is rather different from Euclidean geometry or transformation geometry carried out on paper and raises new problems specific to computer geometry. Another consideration, which applies to any good interactive computer software, is that they give the student the opportunity for independent learning (Ruthven and Hennessy, 2003). There is also what I call ‘learning mathematics by doing it’; the student has to try things out actively and learn what works and what does not. Unfortunately, some students still rely on being able to ask the teacher for help when they meet difficulties, but it does mean that they are more active learners and they can see straight away if what they are trying to do is successful. DGS turns mathematics into a practical science where students can observe, record, manipulate, predict, conjecture, test and theorise. However, firstly it is most important that the teacher becomes familiar with the software both as a tool for their own learning and as a tool for teaching mathematics.

The project

The parts of the geometrical reasoning section of the Y7 programme of study which I covered in the project were: ‘use correct labelling conventions for lines, angles and shapes’; ‘identify parallel and perpendicular lines’; ‘know the properties of triangles and quadrilaterals’; and ‘use reflection, rotation and translation to explore the effect of these transformations on 2D shapes’. I had a target group taught using the computers and a control group taught using pencil and paper methods, covering the same work, to enable the effectiveness of the use of computers to be assessed formally. At the time of this project, I taught two parallel middle sets in Y7, one of which was the target group and the other was the control group, and both groups did the same tests after each section of work. It was important that the control group spent the same amount of time on the topics as the target group. I also gave the control group lots of hands-on activities with cardboard rectangles and paper and scissors so that they and I had as much fun in their lessons as the target group did on the computers. In this way, I was trying to give the control group as much chance of success with geometry as the target group. I analysed the KS2 test results of both groups and found no significant differences between them. I also tested the groups after each section of work. I gave questionnaires to the target group students and interviewed a small number of them.

Making shapes from two congruent right-angled triangles

The first unit of work covered was symmetry of 2D shapes, labelling shapes, lines and angles and congruence. In the first lesson, we used congruent right-angled triangles and put them together to make different shapes. I created a Sketchpad file containing a right-angled triangle which cannot be distorted when dragged. (To do this, the vertices have to be positioned using vectors.) In order to make the shapes, the triangle was transformed using reflection and rotation; I used the digital projector to show the pupils how to do this. They had to take in a lot of information, as this was the first time they had used Sketchpad. I showed them how to construct a kite by choosing the diagonal line of the triangle, marking it as a mirror and reflecting. I showed them how to construct a rectangle by marking the mid-point of the diagonal line and rotating 180°. I asked the pupils to reflect in all three sides in turn and rotate by the mid-points of all three sides. They were quite excited and all created constructions on the screen (figure 1).

Labelling shapes, lines and angles

The second lesson was on labelling shapes, lines and angles. I created a file in Sketchpad containing a quadrilateral divided into a smaller quadrilateral and triangle and with five labelled points (figure 2). I used this sketch to show both classes how to label
shapes, lines and angles and got them to write down the labels of objects I pointed to. The students worked at the computers using the file I had just showed them. Their task was to find the two areas, lengths of lines and sizes of angles. I allowed them to work unaided at first, but eventually told them they would need to highlight the objects and use the ‘measure’ menu. They had to show me when each person in the pair had made some measurements and then swap over. After this, I had devised a shape-fitting exercise, where they had to put various shapes inside a rectangle. My 13-year-old son had made another, harder problem along the same lines (figure 3). (The rectangle and the shapes all have to be constructed using vectors, so that they cannot be dragged out of shape by the students attempting the task.)

The students were intrigued by this youngster inventing problems for them, and attacked it with gusto. When they had solved it, I let them construct their own sketch. They all drew a rectangle and used lines to split it up. They coloured in the different regions, and, although they couldn’t move the shapes about, the students had learned how to construct an interior by choosing vertices in order and using the ‘construct’ menu. So far the students in the target group had learned how to use the line tool to draw shapes, how to reflect and rotate, how to label objects, how to construct interiors and change colours and how to measure areas, lines and angles. They also knew how to put text onto the screen, how to trace a point and how to animate a point.

**Shapes made by reflecting triangles**

For the second part of the project, I prepared a Sketchpad file which consisted of a grid and a mirror line. The task for the pupils in the target group was to draw half of each shape in the following list, put each half next to the mirror line and then complete the shapes using the ‘reflect’ operation from the ‘transform’ menu. The shapes were: isosceles triangle, rectangle, square, kite, rhombus, isosceles trapezium (figure 4).

The pupils worked in pairs and seemed to enjoy the task. They completed it well and were able to see afterwards whether they had correctly drawn half of the shape. In the last part of the lesson, I told the pupils to make accurate drawings of each shape in their books. At first, many of the pupils did not take care to ensure true symmetry and accuracy. After I insisted they do these properly, most managed to be accurate and produced true versions of each shape. Some of them needed help with the rhombus, and I advised them to draw in the diagonals first.
Constructing shapes that have two lines of symmetry

The next two lessons involved the group working on lines of symmetry in shapes. The students were shown how to add a perpendicular line to the previous tile. The original file consisted of a vertical line which I had set up as a line of reflection. (In order to construct the perpendicular to this line, it is necessary to construct a point on the line, highlight the point and the line, go to the ‘construct’ menu and click on ‘construct perpendicular’). Their task was to draw one quarter of a shape, reflect it in the first line of symmetry and then reflect the resultant shape in the second line of symmetry in order to complete the shape. The only quadrilaterals that can be constructed in this way are the square (two ways), rectangle and rhombus. Many of the students needed help to construct the perpendicular line of symmetry. I also found some individuals drawing the shapes using the line tool rather than the ‘transform’ menu! This showed up when I asked them to drag their diagrams; ie, the symmetry was not preserved if the shapes were not generated by reflection. (This ability to preserve symmetry when the object is dragged is a property of DGS.) However, two pupils were able to make a shape and then drag it into one of the other shapes. At the end of the first of the two sessions, I asked them to demonstrate this to the class using my laptop connected to the digital projector. They showed how a rectangle could be turned into a square and how a rhombus could be turned into a square on its corner.

Shapes made by rotating triangles

The next part of the topic was to look at generating shapes by rotating triangles, and the first task I gave the students was to draw various triangles (eg, right-angled, isosceles, equilateral, scalene) on paper and rotate them about the midpoint of a side, with the aid of tracing paper. Some of the students asked if it wouldn’t be better if they used the computer to do it! For the next lesson, I produced a worksheet explaining how to rotate triangles, so that they could do the same thing on computer. However, they only needed to draw one triangle using Sketchpad, and, having rotated it about the midpoint of one side, they could drag the resulting quadrilateral to produce many different shapes (figure 5). I asked them to try to drag the quadrilateral to make specific shapes, and asked why certain ones could not be made. At the end of the task, the class was asked why we could not make a kite or trapezium. The reason is that these shapes do not have rotation symmetry.

In order to construct the objects in DGS, the students have to think more deeply about the geometry than they do if they just draw them on paper. When the students rotated triangles to make quadrilaterals, they had to define the centre of rotation and the angle of rotation. In this way, they had to think about the way a shape is transformed during a rotation as well as the fact that certain quadrilaterals may be constructed by putting two triangles together in a certain way. They were also led to consider general cases by looking at what shapes they could make by dragging a quadrilateral made by reflecting or rotating a triangle. They had to be much more thoughtful about the underlying mathematics when they used Sketchpad. They also had to be precise.

Conclusions

The results of the test given after the first section of the work showed no significant difference between the target group and the control group. However, the results of the second test, given after the second section of the work (carried out in the Spring term), showed that the target group did do better than the control group, and this result was statistically significant. It was interesting to look at the boys and girls separately; the target girls did better than the control girls in both tests; the target boys did less well than the control boys in the first test; however, the target boys did best of all in the second test, showing the most progress of any
group. The sample size was obviously small, but it may be worthwhile someone exploring the gender factor further.

The designing of the computer-based tasks made me think very carefully about what I wanted the students to achieve. I found it was essential to go through a plenary session with the class at the end of a task. The value for the students of using Sketchpad was that, sitting in pairs at a computer, they interacted with the software and learned how to get the computer to produce the desired geometrical objects. This meant that they had to be systematic about the names and labels of geometrical objects and processes. If the computer did not produce what the students wanted, then they had to engage in problem solving to sort this out. They helped each other in their working pairs and also helped other pairs around the classroom. They engaged in discussing and explaining geometrical concepts. During lessons with the computer, the pupils’ conversations tended to involve more mathematics and less inconsequential chatter. In this way, I feel that their understanding of the concepts was better than if they had learned geometry through paper-based tasks. However, mixing computer work with paper-based tasks is probably the best way to ensure the students can access the mathematics and enhance their understanding further.

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References
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Goldstein, R. (2002) Teaching the computer, Micronath, 18/1, pp 5-7

Notes
1 SMILE (Secondary Mathematics Individualised Learning Experiment) was run by what was the Inner London Education Authority. (See page 44 for an historical perspective on this.)
2 This project was my dissertation for my masters degree in computer-enhanced mathematics education at Napier University.
3 The software used in the project was Geometer’s Sketchpad and Sue’s files (and worksheets) are available at www.atm.org.uk/ntt/.